Elastic, Strings and Springs

Questions

Q1.

A spring of natural length *a* has one end attached to a fixed point *A*. The other end of the spring is attached to a package *P* of mass *m*.
The package *P* is held at rest at the point *B*, which is vertically below *A* such that *AB* = 3*a*.
After being released from rest at *B*, the package *P* first comes to instantaneous rest at *A*.
Air resistance is modelled as being negligible.

By modelling the spring as being light and modelling *P* as a particle,

(a) show that the modulus of elasticity of the spring is 2*mg*

(5)

(b) (i) Show that *P* attains its maximum speed when the extension of the spring is $\frac{1}{2}a$

(ii) Use the principle of conservation of mechanical energy to find the maximum speed, giving your answer in terms of *a* and *g*.

(6)

In reality, the spring is not light.

(c) State one way in which this would affect your energy equation in part (b).

(1)

(Total for question = 12 marks)

Q2.

A light elastic string with natural length l and modulus of elasticity kmg has one end attached to a fixed point A on a rough inclined plane. The other end of the string is attached to a package of mass m.

The plane is inclined at an angle θ to the horizontal, where $\tan\theta = \frac{5}{12}$

The package is initially held at A. The package is then projected with speed $\sqrt{6gl}$ up a line of greatest slope of the plane and first comes to rest at the point B, where $AB = 3l$.

The coefficient of friction between the package and the plane is $\frac{1}{4}$

By modelling the package as a particle,

(a) show that $k = \frac{15}{26}$

(6)

(b) find the acceleration of the package at the instant it starts to move back down the plane from the point B.

(5)

(Total for question = 11 marks)

Q3.

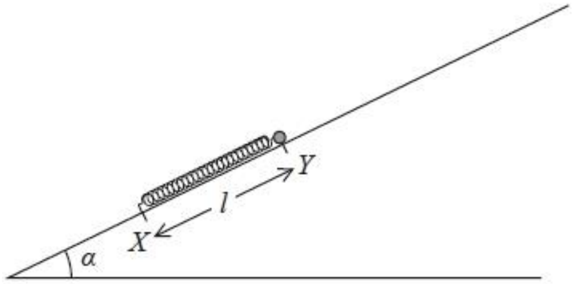

Figure 2

A light elastic spring has natural length 3*l* and modulus of elasticity 3*mg*.

One end of the spring is attached to a fixed point *X* on a rough inclined plane.

The other end of the spring is attached to a package *P* of mass *m*.

The plane is inclined to the horizontal at an angle *α* where $\tan\alpha = \frac{3}{4}$

The package is initially held at the point *Y* on the plane, where *XY* = *l*. The point *Y* is higher than *X* and *XY* is a line of greatest slope of the plane, as shown in Figure 2.

The package is released from rest at *Y* and moves up the plane.

The coefficient of friction between *P* and the plane is $\frac{1}{3}$

By modelling *P* as a particle,

(a) show that the acceleration of *P* at the instant when *P* is released from rest is $\frac{17}{15}g$

(5)

(b) find, in terms of *g* and *l*, the speed of *P* at the instant when the spring first reaches its natural length of 3*l*.

(6)

(Total for question = 11 marks)

Q4.

The ends of a light elastic string, of natural length 0.4 m and modulus of elasticity λ newtons, are attached to two fixed points *A* and *B* which are 0.6 m apart on a smooth horizontal table. The tension in the string is 8 N.

(a) Show that $\lambda = 16$

(3)

A particle *P* is attached to the midpoint of the string. The particle *P* is now pulled **horizontally** in a direction perpendicular to *AB* to a point 0.4 m from the midpoint of *AB*. The particle is held at rest by a **horizontal** force of magnitude *F* newtons acting in a direction perpendicular to *AB*, as shown in Figure 5 below.

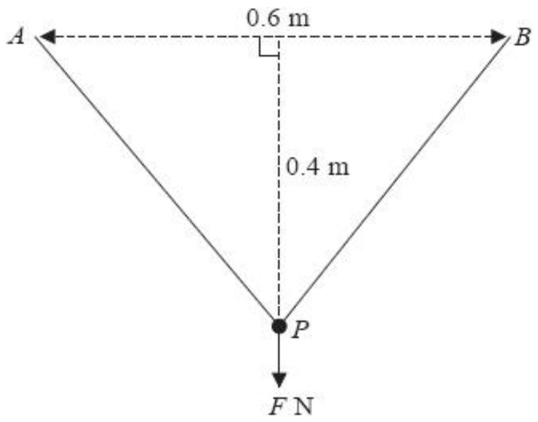

Figure 5

(b) Find the value of *F*.

(4)

The particle is released from rest. Given that the mass of *P* is 0.3 kg,

(c) find the speed of *P* as it crosses the line *AB*.

(6)

(Total for question = 13 marks)

Q5.

Unless otherwise indicated, whenever a numerical value of g is required, take g = 9.8 ms^{-2} and give your answer to either 2 significant figures or 3 significant figures.

A particle P of mass m is attached to one end of a light elastic string of natural length a and modulus of elasticity $3mg$.

The other end of the string is attached to a fixed point O on a ceiling.

The particle hangs freely in equilibrium at a distance d vertically below O.

(a) Show that $d = \frac{4}{3}a$.

(3)

The point A is vertically below O such that $OA = 2a$.

The particle is held at rest at A, then released and first comes to instantaneous rest at the point B.

(b) Find, in terms of g, the acceleration of P immediately after it is released from rest.

(3)

(c) Find, in terms of g and a, the maximum speed attained by P as it moves from A to B.

(5)

(d) Find, in terms of a, the distance OB.

(3)

(Total for question = 14 marks)

Q6.

A particle P, of mass m, is attached to one end of a light elastic spring of natural length a and modulus of elasticity kmg.

The other end of the spring is attached to a fixed point O on a ceiling.

The point A is vertically below O such that $OA = 3a$

The point B is vertically below O such that $OB = \frac{1}{2}a$

The particle is held at rest at A, then released and first comes to instantaneous rest at the point B.

(a) Show that $k = \frac{4}{3}$

(3)

(b) Find, in terms of g, the acceleration of P immediately after it is released from rest at A.

(3)

(c) Find, in terms of g and a, the maximum speed attained by P as it moves from A to B.

(6)

(Total for question = 12 marks)

Mark Scheme – Elastic, Strings and Springs

Q1.

Question	Scheme	Marks	AOs
(a)	EPE at $A=\frac{\lambda a^2}{2a}$ or EPE at $B=\frac{\lambda(2a)^2}{2a}$	M1	2.1
	Form work-energy equation:	M1	3.3
	$\frac{\lambda a^2}{2a}+mg\times 3a=\frac{\lambda(2a)^2}{2a}$ $\left(\frac{\lambda a}{2}+3mga=2\lambda a\right)$	A1 A1	1.1b 1.1b
	$3mg=\frac{3\lambda}{2}$ $\Rightarrow$ $\lambda=2mg$ *	A1*	2.2a
		(5)	
(b)	Extension at equilibrium:	M1	2.1
	$\frac{2mgx}{a}=mg$ $\Rightarrow$ $x=\frac{a}{2}$ *	A1*	1.1b
	Alternative for the first M1A1:		
	Use the work-energy equation to obtain $\frac{dV^2}{dx}$ and set the derivative equal to zero	M1	
	$\frac{1}{a}\times 2x-1=0\Rightarrow x=\frac{a}{2}$	A1	
	Use work-energy equation to find max speed:	M1	3.4
	$\frac{2mgx^2}{2a}+mg\times(2a-x)+\frac{1}{2}mV^2=\frac{2mg(2a)^2}{2a}$ $\left(\frac{ag}{4}+\frac{3ag}{2}+\frac{1}{2}V^2=4ag\right)$	A1 A1	1.1b 1.1b
	$V=3\sqrt{\frac{ag}{2}}$	A1	2.2a
		(6)	

(c)	e.g. for B1 Need to include the GPE of the spring The extension of the spring at equilibrium will be different The spring will have KE You would need to include the KE of the spring in the energy equation You would need to include the GPE of the spring in the energy equation The GPE of the system changes It would take work to raise the spring so the package would have less KE If the spring has mass then GPE of the spring would need to be included	B1	3.5b
		(1)	
		(Total 12 Marks)	

Notes	
(a) M1	Correct method for EPE seen or implied Need something of the form $\frac{1}{2}kx^2$ where $k = \frac{\lambda}{a}$ Must be using the formula for EPE correctly at least once
M1	Require all terms. Dimensionally correct. Condone their EPE. Condone sign errors
A1 A1	Unsimplified equation with at most one error. A repeated error in EPE formula is one error Correct unsimplified equation.
A1*	Obtain **given answer** from correct working
(b) M1	Use correct method for tension to find the extension at equilibrium. Need to see the formula for tension **used**. **Allow verification** with an appropriate conclusion If they use SHM they must use $F = ma$ to prove that P is moving with SHM, otherwise 0/2.
A1*	Correct answer from correct work Allow verification **with an appropriate conclusion**
Alt:M1	Or an equivalent method for finding the turning point of a quadratic
Alt:A1*	Correct answer from correct work
M1	Use given x to form work-energy equation. Need all terms, and dimensionally correct. Condone sign errors. Accept with values of λ and x not substituted
A1 A1	Unsimplified equation with at most one error. Need given λ and given x substituted at some point. A repeated error in the formula for EPE is one error. Correct unsimplified equation with given λ and given x substituted at some point
A1	Use correct method for tension to find the extension at equilibrium. Any equivalent form. $2.1\sqrt{ag}$ or better

(c) B1	Any valid response. B0 if answer includes an additional incorrect factor. Must be specific e.g. not just "the GPE changes", but the GPE of the system changes is OK. Must relate to an effect on the energy equation E.g. for B0 The extension changes *AB* will increase The tension/energy/GPE/work done etc would increase The KE/GPE/EPE/acceleration/extension/velocity changes The mass of the spring would drag down and the EPE would change The EPE/KE/GPE etc would be variable
	There would be tension in the spring as well It has weight The velocity would decrease as energy is converted

Q2.

Question	Scheme	Marks	AOs
(a)	Work done against friction $=3l\times\mu mg\cos\theta\ \left(=\frac{9mgl}{13}\right)$	B1	3.4
	Gain in EPE $=\frac{kmg\times 4l^2}{2l}\quad(=2kmgl)$	B1	3.4
	Gain in GPE $=mg\times 3l\sin\theta\ \left(=\frac{15mgl}{13}\right)$	B1	3.4
	Work energy equation:	M1	2.1
	$\frac{1}{2}m\times 6gl=\frac{9mgl}{13}+2kmgl+\frac{15mgl}{13}$	A1	1.1b
	$2k=3-\frac{24}{13}=\frac{15}{13},\quad k=\frac{15}{26}\quad *$	A1*	2.2a
		(6)	
(b)	Tension in the string at *B*: $\frac{\frac{15}{26}mg\times 2l}{l}\ \left(=\frac{15mg}{13}\right)$	B1	3.1a
	Equation of motion: tension + component of weight − friction = *ma*	M1	3.3
	$\frac{15mg}{13}+mg\sin\theta-\frac{1}{4}mg\cos\theta=ma$ $\left(mg\left(\frac{15}{13}+\frac{5}{13}-\frac{3}{13}\right)=ma\right)$	A1 A1	1.1b 1.1b
	$a=\frac{17g}{13}$	A1	1.1b
		(5)	
		(11 marks)	

Notes:	
(a)B1 B1 B1	Use model to obtain one correct term Use model to obtain two correct terms Use model to obtain three correct terms
M1	Work-energy equation. Need all terms and no extras. Dimensionally correct. Condone sign errors and sin/cos confusion.
A1	Correct unsimplified equation
A1*	Obtain given result from correct working
	NB: The use of *suvat* equations is not a valid alternative method because the acceleration is not constant
(b) B1	Correct unsimplified expression for the tension in the string
M1	Equation of motion. Need all terms and no extras. Condone sign errors and sin/cos confusion Allow with T or their T
A1 A1	Unsimplified equation with at most one error Correct unsimplified equation
A1	Exact answer or accept 12.8 or 13 $\left(\text{m s}^{-2}\right)$

Q3.

Question	Scheme	Marks	AOs
(a)	Thrust in the spring $=\dfrac{3mg2l}{3l}\quad(=2mg)$	B1	2.1
	Equation of motion:	M1	3.3
	$2mg-mg\sin\alpha-\dfrac{1}{3}mg\cos\alpha=ma$ $\left(2mg-\dfrac{3mg}{5}-\dfrac{4mg}{15}=ma\right)$	A1ft A1ft	1.1b 1.1b
	$a=\dfrac{17g}{15}$ *	A1*	2.2a
		(5)	
(b)	Initial EPE $=\dfrac{3mg4l^2}{2\times 3l}\quad(=2mgl)$ Gain in GPE $=mg2l\sin\alpha\quad\left(=\dfrac{6}{5}mgl\right)$ Work done against friction $=\dfrac{1}{3}mg\cos\alpha\times 2l\quad\left(=\dfrac{8}{15}mgl\right)$	B1 B1 B1	3.4 3.4 3.4
	Work-energy equation:	M1	3.1a
	$\dfrac{1}{2}mv^2+\dfrac{2}{3}mgl\cos\alpha+2mgl\sin\alpha=2mgl$	A1	1.1b
	$v=\sqrt{\dfrac{8gl}{15}}$	A1	1.1b
		(6)	
		(11 marks)	

Notes:	
(a) B1	Correct unsimplified expression for the thrust
M1	Equation of motion. All required terms and no extras. Dimensionally correct. Condone sign errors and sin/cos confusion
A1ft A1ft	Unsimplified equation with at most one error (in T or their T) Correct unsimplified equation (in T or their T)
A1*	Obtain given result from correct working
(b) B1 B1 B1	Use model to obtain one correct term Use model to obtain two correct terms Use model to obtain three correct terms

M1	All required terms and no extras. Dimensionally correct. Condone sign errors and sin/cos confusion.
A1	Correct unsimplified equation
A1	Accept $0.73\sqrt{gl}$

Q4.

Question Number	Scheme	Marks
(a)	$8=\frac{\lambda\times 0.20}{0.40}$	M1A1
	$\lambda=16$ *	A1cso (3)
(b)	Length of string = 1 m or 100cm	
	$T=\frac{\lambda\times 0.6}{0.4}$, $=24$ (or use half string)	M1,A1
	$2T\cos\theta=F$	M1
	$F=2\times 24\times\frac{4}{5}=38.4$, $\frac{192}{5}$ or $38\frac{2}{5}$	A1 (4)
(c)	Initial EPE $=\frac{16\times 0.6^2}{2\times 0.4}\left(=\frac{36}{5}\right)$ Final EPE $=\frac{16\times 0.2^2}{2\times 0.4}\left(=\frac{4}{5}\right)$	B1 (either)
	$\frac{16\times 0.6^2}{2\times 0.4}-\frac{16\times 0.2^2}{2\times 0.4}=\frac{1}{2}0.3v^2$	M1A1A1
	$0.3v^2=40\left(0.6^2-0.2^2\right)$	
	$v=6.531...$ Accept $6.5\left(\text{m s}^{-1}\right)$ or better or exact value $8\sqrt{\frac{2}{3}}\ \left(\text{m s}^{-1}\right)$	dM1A1cso (6)
		[13]

(a)

M1 Attempt Hooke's Law using the whole string or a half string.

A1 Correct equation.

A1cso Correct **given** value of λ obtained with no errors seen.

(b)

M1 Use Hooke's Law with the new longer length for the string or half string. λ must be 16, but length need not be correct but use of 0.2 for extension of full string or 0.1 for extension of half string scores M0.

A1 Obtain $T = 24$

M1 Resolve parallel to F or in another direction which gives an equation connecting T and F.

A1 Obtain the correct value of F

(c)

B1 Correct initial or final EPE with one string ($l = 0.4$) or two half strings ($l = 0.2$)

M1 Attempt an energy equation with the difference of 2 EPE terms and a KE term. The EPE terms must be of the form $k\frac{\lambda x^2}{l}$.

A1A1 Deduct one mark per error. (A1A1, A1A0 or A0A0)

dM1 Solve for v. Depends on the previous M mark.

A1cso Correct value of v, min 2 sf or exact value.
Energy terms wrong way round in the equation will lose this mark even if modulus sign inc here.

NB If the energy terms are subtracted the wrong way round, max score is B1M1A1A0M1A0

Q5.

Question	Scheme	Marks	AOs
(a)	In equilibrium $\Rightarrow$ no resultant vertical force.	M1	2.1
	$\frac{3mgx}{a} = mg$	A1	1.1b
	$x = \frac{a}{3}$, $d = \frac{4}{3}a$ *	A1*	2.2a
		(3)	
(b)	Equation of motion.	M1	3.1a
	$\frac{3mga}{a} - mg = m\ddot{x}$	A1	1.1b
	$\ddot{x} = 2g$	A1	1.1b
		(3)	
(c)	Max speed at equilibrium position	B1	3.1a
	Work energy & use of EPE $= \frac{\lambda x^2}{2a}$.	M1	3.1a
	$\frac{3mga^2}{2a} = \frac{3mg\left(\frac{a}{3}\right)^2}{2a} + \frac{1}{2}mv^2 + mg\frac{2a}{3}$	A1 A1	1.1b 1.1b
	$\frac{1}{2}v^2 = ga\left(\frac{3}{2} - \frac{1}{6} - \frac{2}{3}\right) = \frac{2}{3}ga$, $v = \sqrt{\frac{4ga}{3}}$	A1	1.1b
		(5)	
(d)	At max ht. KE = 0. EPE lost = GPE gained	M1	3.1a
	$\frac{3mga^2}{2a} = mgh$	A1	1.1b
	$OB = \frac{a}{2}$	A1	1.1b
		(3)	
			(14 marks)

Question continued	
Notes:	
(a)	
M1:	Use $T = \dfrac{\lambda x}{a}$ to form equation for equilibrium
A1:	Correct unsimplified equation
A1*:	Requires sufficient working to justify **given answer** plus a 'statement' that the required result has been achieved.
(b)	
M1:	Use $T = \dfrac{\lambda x}{a}$ to form equation of motion.
	Need all 3 terms. Condone sign errors
A1:	Correct unsimplified equation
A1:	cao
(c)	
B1:	Seen or implied
M1:	Form work-energy equation. All 4 terms needed. Condone sign errors
A1:	Correct unsimplified equation A1A1
	One error in the equation A1A0
A1:	cao
(d)	
M1:	Form energy equation
A1:	Correct unsimplified equation
A1:	cao

Q6.

Question	Scheme	Marks	AOs	Notes
(a)	From A to B EPE lost = GPE gained	M1	2.1	Use conservation of energy with EPE $=\frac{\lambda x^2}{2a}$. (Condone EPE $=\frac{\lambda x^2}{a}$ here). All three terms required. Must be dimensionally correct. Condone sign errors.
	$\frac{kmg\times 4a^2}{2a}-\frac{kmg\times\frac{a^2}{4}}{2a}=mg\times\frac{5a}{2}$	A1	1.1b	Correct unsimplified equation in k
	$k=\frac{4}{3}$ *	A1*	2.2a	Derive given result from correct working.
		(3)		
(b)	At A, equation of motion:	M1	3.1a	Use $T=\frac{\lambda x}{a}$ and N2L to form equation of motion. All terms required. Dimensionally correct. Condone sign errors
	$(T-mg=)\frac{4mg\times 2a}{3a}-mg=m\times$	A1	1.1b	Correct unsimplified equation
	$\Rightarrow$ acceleration $=\frac{5g}{3}$	A1	1.1b	Correct only ISW. Condone 1.7g or better. Accept + / -
		(3)		

Question	Scheme	Marks	AOs	Notes
(c)	Max speed at equilibrium position	M1	3.1a	Maximum speed at equilibrium seen or implied, and correct method to find e
	$\frac{4mge}{3a} = mg,$ $e = \frac{3a}{4}$	A1	1.1b	Correct e
				Alternative: form energy equation for movement through a height of h and differentiate v^2 wrt h to find h for max v M1 $h = \frac{5a}{4}$ A1
	Forms equation using conservation of energy	M1	3.1a	Form energy equation for movement from A to equilibrium position. Need all 4 terms. Correct form for EPE. Dimensionally correct. Condone sign errors. Allow in a, g and e (with e defined)
	$\frac{4mg \times 4a^2}{3 \times 2a} = \frac{4mg \times \frac{9a^2}{16}}{3 \times 2a} + \frac{1}{2}mv^2$	A1ft A1ft	1.1b 1.1b	Unsimplified equation in their e with at most one error Correct unsimplified equation (using their e) for v
	$v = \frac{5}{2}\sqrt{\frac{ga}{3}}$	A1	1.1b	Any equivalent form. Accept $1.44\sqrt{ag}$ or $1.4\sqrt{ag}$
		(6)		
				SHM is not on this specification, but you might see some candidates using it. See over for SHM alternative for parts (b) and (c)

	At equilibrium, $\frac{4mge}{3a} = mg,\ e = \frac{3a}{4}$			They need to start by showing that they have SHM in order tto justify using the standard results. No marks scored for this at this stage.
	Equation of motion: $mg - \frac{4mg}{3a}(e+x) = m\ddot{x}$, so $\ddot{x} = -\frac{4g}{3a}x$ Hence SHM			
	(b) Use of $x = \frac{5a}{4}$ and their ω^2	M1		Substitute to find acceleration
	$\ddot{x} = -\frac{4g}{3a} \times \frac{5a}{4} = -\frac{5g}{3},\ \lvert\ddot{x}\rvert = \frac{5g}{3}$	A1		Correct only ISW. Condone 1.7g or better
		(2)		
	(c) $\frac{4mge}{3a} = mg,$	M1		This work now scores the two marks provided it is used in part (c)
	$e = \frac{3a}{4}$	A1		
	Use of $v_{max} = \omega a$	M1		Correct method to find max v
	$v_{max} = \sqrt{\frac{4g}{3a}} \times \frac{5a}{4}$	A2ft		Follow their e and ω
	$v_{max} = \frac{5}{2}\sqrt{\frac{ga}{3}}$	A1		Any equivalent form. Accept $1.44\sqrt{ag}$ or $1.4\sqrt{ag}$
		(6)		
	(Total 12 marks)			